Dear Ryan:

I have studied the subject of stability in masonry walls for some time. I believe that there are problems with the MSJC treatment of stability in masonry compression members. Let me offer you my insights.

<u>Unreinforced Masonry</u>

The basic treatment of stability in the entire MSJC document is based on the solution for a compression member that

(1) is made of a material that is linearly elastic in compression,
(2) has no tensile strength,
(3) is compressed eccentrically, and
(4) has eccentricity that is constant over the height of the member.

The solution for this problem is given by Eq. (2-11) for P_e, and a basic factor of safety equal to 4 is used to define the maximum axial load that an unreinforced member can carry to avoid instability. This solution was reported by Yokel (1971).

For Allowable Stress Design (ASD), an allowable axial compression stress, F_a, is also defined for unreinforced masonry by either Eq. (2-12), for stocky members (h/r <99), or by Eq. (2-13), for relatively slender members (h/r>99). The second one of these formulas, Eq. (2-13), was derived from Eq. (2-11) assuming an eccentricity, e, equal to $0.1t$, where t is the thickness of the member, and a modulus of elasticity, E_m, equal to $1000f'_m$. I will refer to Eq. (2-13) as the 'elastic buckling' branch because it arises from Eq. (2-11) which is an elastic solution. The stress function given by Eq. (2-12) defines a transition from pure compression ($F_a = f'_m/4$) to the value for F_a given by the 'elastic buckling' branch (Eq. (2-13)) at a slenderness ratio, h/r, equal to 99. I will refer to this formula as the 'inelastic buckling' branch because it transitions to pure compression (i.e., material crushing).

The allowable axial load for a member with eccentricity, e, smaller than $0.1t$, will most likely not be controlled by Eq. (2-11), because the formula for F_a has an implicit eccentricity that is larger than the actual value. Similarly, for members with large values for eccentricity (i.e., exceeding $0.1t$), Eq. (2-11) will, most likely, control the maximum allowable axial load. I say "most likely" because the implicit value for E_m in Eq. (2-12) and (2-13) is not usually equal to $1000f'_m$.

<u>Influence of Bending in Unreinforced Compression Members</u>

For a compression member with bending due to lateral loading (out-of-plane lateral loading in the case of a wall), <u>the MSJC document does not provide treatment of the detrimental effect of the moment produced by these loads</u>. I refer to it as a <u>detrimental effect</u> because such bending introduces flexural tension stresses that have the potential to crack the member section. This cracking undermines the effective moment of inertia, I, of the member, and thus its resistance to buckling. In my opinion, this omission can lead to highly dangerous situations, and I have warned the MSJC about the problem. For more on my concerns on this issue, see Schultz and Mueffelman (2003a).

The statement in the commentary about the eccentricity, e, is correct in the strictest sense. It is the actual value due to axial load placement, and not a virtual value arising from bending (i.e., M/P), because that is how Yokel (1971) defined it in the derivation from which the MSJC obtained Eq. (2-11). However, Yokel's solution can be extended to include bending by means of a flexural eccentricity term, M/P, if the denominator is taken as the axial load at the moment of instability (i.e., the buckling load). I re-derived Yokel's solution using such a virtual eccentricity, defined as $e_f = M/P$ (see Schultz and Mueffelman, 2003b). I also developed an approximate solution for the buckling axial load under the influence of bending, P_{ef}, which is given in the following

$$P_{ef} = \frac{\pi^2 E_m I_n}{h_e^2}\left[1-0.577\left(\frac{e_a+\lambda e_f}{r}\right)\right]^3$$

The formula above follows the format of Eq. (2-11) except that, in addition to the actual eccentricity (e_a), it includes an eccentricity term for the effect of bending from lateral (out-of-plane) loading. This term ($\lambda \cdot e_f = \lambda \cdot M/P$) includes an additional factor, λ, that is controlled by the shape of the first-order moment diagram. My research group has also conducted tests to verify this solution (see Bean Popehn et al. 2008).

The solution above gives rise to an interaction between moment, M, and axial load, P, at instability which is highly nonlinear. To solve this interaction for P, given a value for M, is messy because you have to solve a 4th order polynomial. It is far easier to solve for M, given a value for P. In a more recent paper (Schultz, 2011), I propose an approximate solution for P using 2nd order polynomials which is given by

$$P_{ef} = 0.422 P_e\left[1-\sqrt{1-\frac{5.47\left(\frac{\lambda M_w}{P_e r}\right)}{\left(1-0.577\frac{e_a}{r}\right)}}\right]$$

where P_e is calculated using Eq. 2-11, and M_w is the first-order moment from lateral loading. The lateral load moment, M_w, cannot exceed the largest moment in the interaction curve, M_{mp}, which is given by

$$M_{mp} = \frac{0.1828 P_e r}{\lambda}\left(1-0.577\frac{e_a}{r}\right)$$

The formula for P_{ef} above is one of two, because there are two axial load-bending moment (M-P) combinations that satisfy the stability criterion. For cases of bending moment from environmental effects (i.e., wind or seismic loads), the formula above should be used because M_w is highly variable.

Reinforced Masonry

You are correct in noting that for reinforced masonry in the Allowable Stress Design, there is nothing in the MSJC regarding stability other than the use of the buckling factors previously derived for inelastic and elastic buckling. As you note, there is no consideration of second-order effects as there is in Eq. (2-11) for unreinforced, eccentrically-compressed masonry walls.

I am further concerned that when a masonry member with concentrically placed reinforcement (e.g., a wall with a single row of bars at the center of the wall) is used, that cracking of the member results in large reductions in both the moment of inertia, I, and the buckling load. Large reductions occur because the effective thickness of the member is halved (by virtue of the placement of the reinforcement), and moment of inertia is proportional to the effective thickness raised to the 4th power. Moreover, masonry typically has low reinforcement ratios, and this feature makes the difference between uncracked and cracked stiffness even larger.

For a typical grouted masonry wall with a single row of reinforcing bars at the center of the wall, the ratio between cracked section and gross section moments of inertia can be as small as 0.01 (and as large as 0.25). The corresponding reductions in buckling capacity are certainly not included in the MSJC treatment of reinforced masonry compression member. ACI 318, on the other hand, addresses such differences in stiffness by means of its provisions for the treatment of slender columns.

Summary

In summary, (1) the MSJC code currently includes no consideration of bending in the stability of URM compression members, and (2) there is no systematic treatment of stability in ASD for reinforced masonry compression members.

References

Yokel, F.Y. (1971). “Stability and load capacity of members with no tensile strength.” *Journal of Structural Engineering*, ASCE, Vol. 97, No. ST7, pp. 1913-1926.

Schultz, A.E., Mueffelman, J.G. (2003a) “Design considerations for stability of transversely loaded URM walls.” *TMS Journal*, The Masonry Society, Vol. 21, No. 1, pp. 41–53.

Schultz, A.E., Mueffelman, J.G. (2003b). “Elastic stability of URM walls under transverse loading.” *TMS Journal*, The Masonry Society, Vol. 21, No. 1, pp. 31–40.

Bean Popehn, J.R., Schultz, A.E., Lu, M., Stolarski, H.K., Ojard, N.J. (2008). “Influence of transverse loading on the stability of slender unreinforced masonry walls.” *Engineering Structures*, Elsevier, Vol. 30, No. 10, pp. 2830-2839.

Schultz, A.E. (2011). “Solutions for Critical Axial Loads in Eccentrically Compressed and Laterally Loaded Slender URM Members,” *Proceedings*, 11th North American Masonry Conference, Minneapolis, MN, June 5-8.