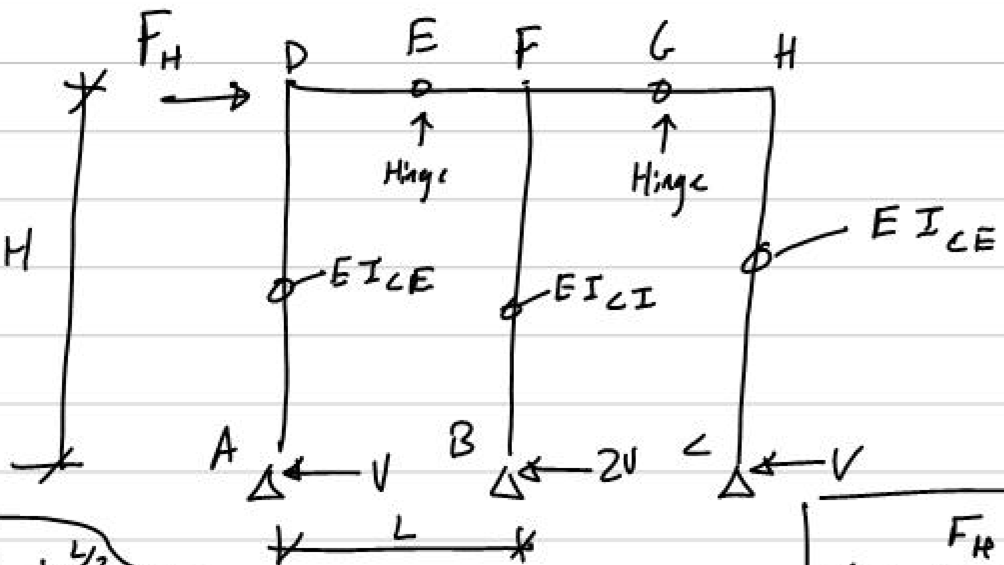
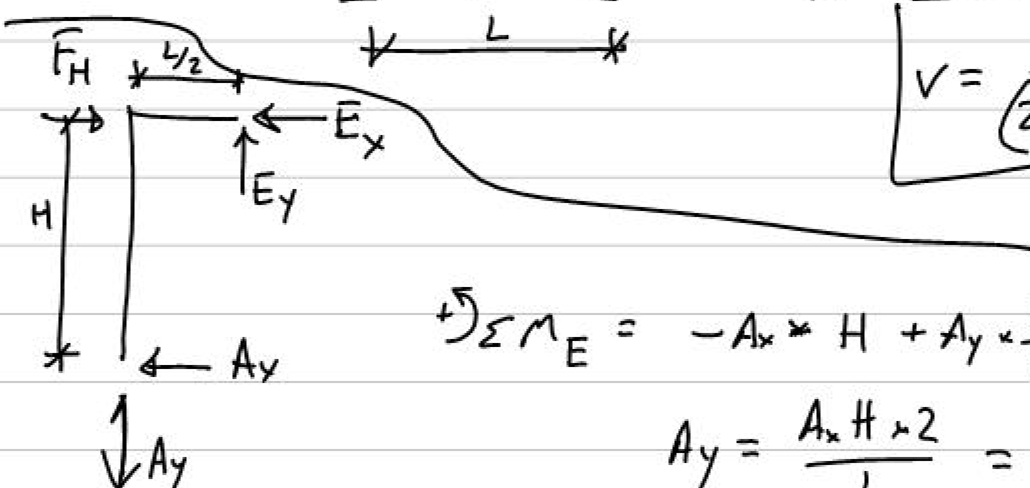
Portal Method

- Moments & Deflection Due to Bending



$$\boxed{V = \frac{F_H}{(2N_{col} - 2)}}$$



$$\circlearrowleft^{+} \Sigma M_E = -A_x * H + A_y \times \frac{L}{2} = 0$$

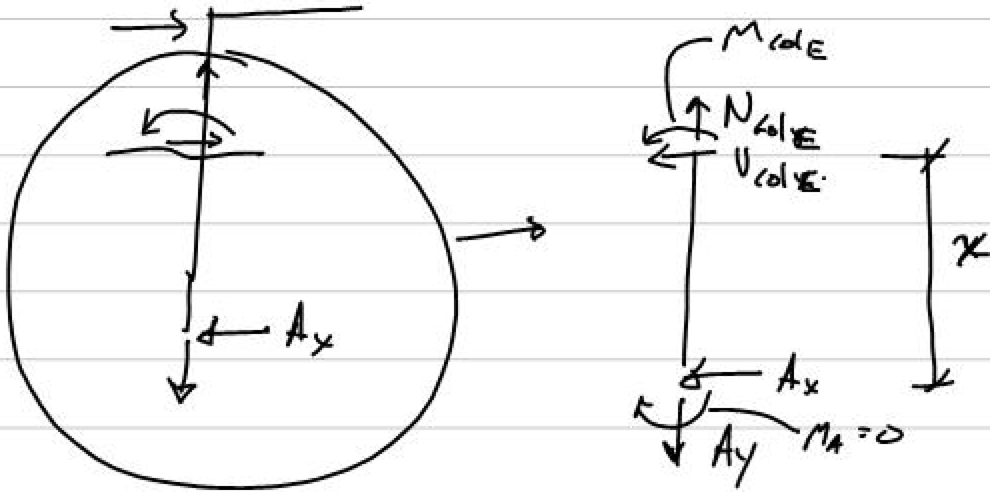
$$A_y = \frac{A_x H \times 2}{L} = \frac{2VH}{L}$$

$$\Sigma F_x = -A_x + F_H - E_x = 0$$

$$E_x = F_H - A_x = F_H - V$$

$$\Sigma F_y = A_y = E_y = \frac{2VH}{L}$$

Internal Force Diagram



$$+\circlearrowleft \Sigma M = M_{colE} - A_x x = 0$$

$$M_{colE} = A_x x = V x$$

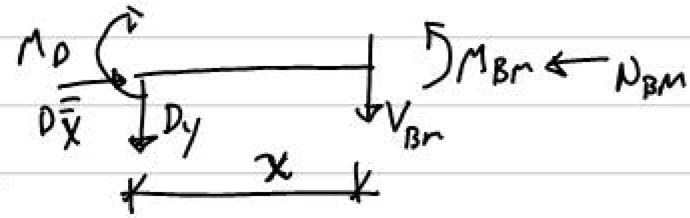
$$\Sigma V = V_{colE} + A_x \Rightarrow V_{colE} = A_x = V$$

$$\Sigma N = N_{colE} = A_y = \frac{2VH}{L}$$

$$M_D = VH$$

$$D_y = \frac{2VH}{L}$$

$$D_x = F_H - V$$

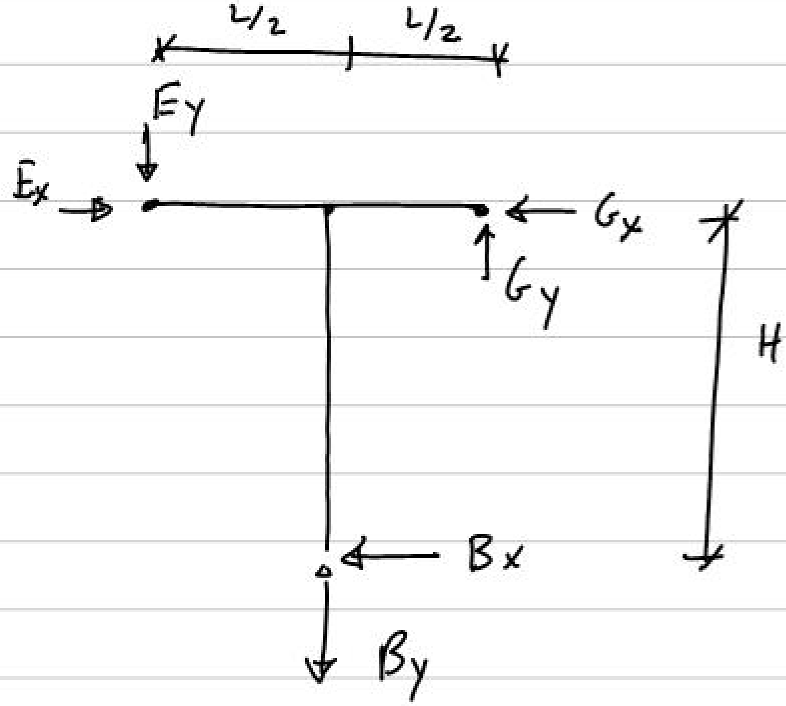


$$\Sigma M = M_{Bm} - M_D + D_y x = 0$$

$$M_{Bm} = M_D - D_y x = VH - \frac{2VH}{L} x$$

$$\Sigma V \Rightarrow V_{Bm} = D_y = \frac{2VH}{L}$$

$$\Sigma N = D_x - N_{Bm} \Rightarrow N_{Bm} = D_x = F_H - V$$



$$\circlearrowleft \Sigma M_G = L \times E_y + L/2 \times B_y - B_x H = 0$$

$$B_y = \frac{(B_x H - E_y L)2}{L} = \cancel{\frac{\left(2VH - \frac{2VH}{L} \times L\right)2}{L}} \rightarrow 0$$

$$\Sigma F_x = E_x - B_x - G_x = 0$$

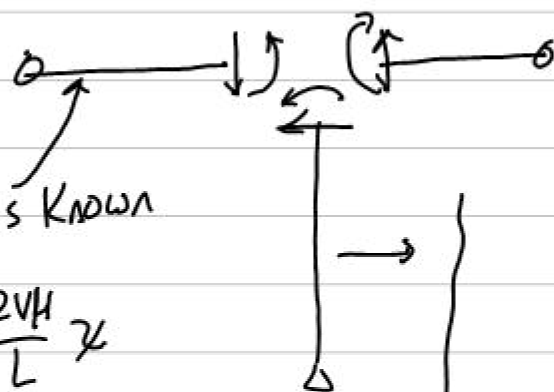
$$G_x = E_x - B_x = F_H - V - 2V = F_H - 3V$$

$$\Sigma F_y = -E_y - \cancel{B_y} \rightarrow 0 + G_y = 0$$

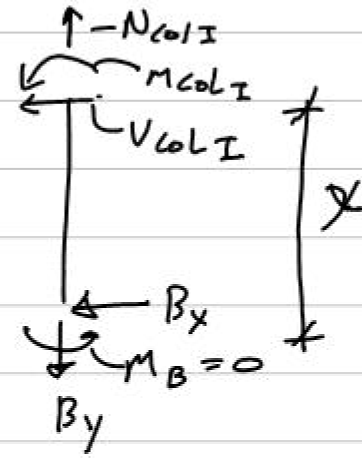
$$E_y = G_y = \frac{2VH}{L}$$

Internal Forces

Approx. Method Moment is <u>Not</u> "distributed" By stiffness



moment is Known



$$M_{BM} = VH - \frac{2VH}{L}x$$

@ $x = L$

$$M_{BM} = -VH$$

$$\Sigma M = M_{col\,I} - B_x x = 0$$

$$M_{col\,I} = B_x x = 2Vx$$

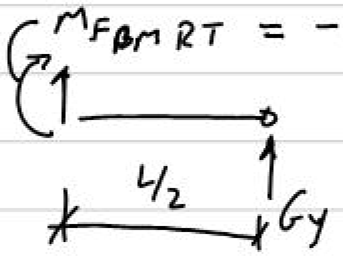
$$M_{@\cancel{x}=H} = 2VH$$

$$M_{F_{col}} = 2VH$$

$$\Sigma N = B_y = N_{col\,I} = 0$$

$$\Sigma V = V_{col\,I} = B_x = 2V$$

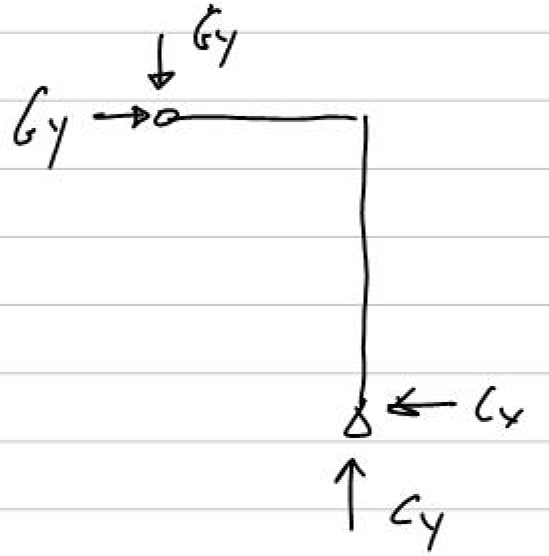
$$M_{F_{BM\,RT}} = -VH + 2VH = VH$$



check

Moment is same as other Beam

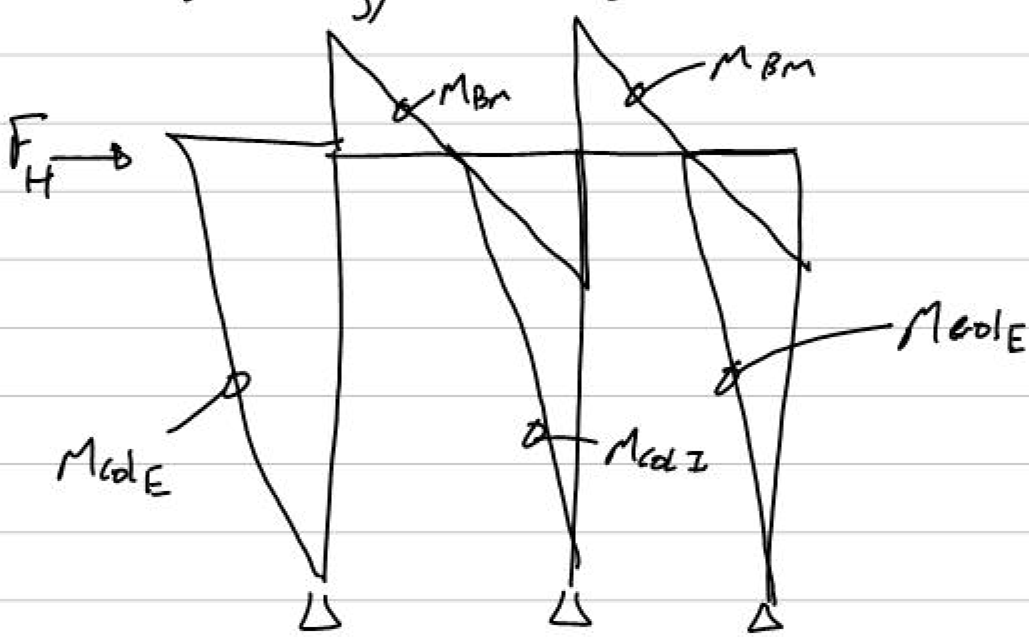
$$M_F = G_y \times L/2 = \frac{\cancel{2}VH}{\cancel{L}} \cdot \frac{\cancel{L}}{\cancel{2}} = VH \quad \text{ok}$$



→ Same as 1'st Ex. Column.

Now Find Deflection of Frame due to Bending

- use energy methods.



$$M_{col\,E} = Vx = \frac{F_H}{(2N_{col}-2)}x$$

$$M_{col\,I} = 2Vx = 2\left[\frac{F_H}{2N_{col}-2}\right]x$$

$$M_{Bm} = VH - \frac{2VH}{L}x \Rightarrow \frac{F_H}{(2N_{col}-2)}H - \frac{2F_H H}{L(2N_{col}-2)}x$$

$$\Delta = \int \frac{M\,m}{EI}$$

M — Real, F_H

m — Virtual, $F_H = 1$

Lets use $n = (2N_{col}-2)$

So

single Ext. col. $\rightarrow$

$$\Delta_{col\,E} = \frac{1}{EI_{cE}}\int_0^H \frac{F_H}{n}x\left(\frac{1}{n}\right)x = \int_0^H \frac{F_H}{n^2}x^2 = \frac{F_H x^3}{3n^2\,EI_{cE}} = \boxed{\frac{F_H H^3}{3n^2\,EI_{cE}}}$$

Single Int. Colum $\rightarrow$

$$\Delta_{col\,I} = \frac{1}{EI_{cI}}\int_0^H \frac{2F_H}{n}x\left(\frac{2}{n}x\right)$$

$$= \frac{1}{EI_{cI}}\int_0^H \frac{4F_H}{n^2}x^2 = \frac{4F_H}{3n^2}x^3 = \boxed{\frac{4F_H H^3}{3n^2\,EI_{cI}}}$$

$$\Delta_{Bm} = \frac{1}{EI_b}\int_0^L \left(\frac{F_H H}{n} - \frac{2F_H H}{Ln}x\right)\left(\frac{H}{n} - \frac{2H}{Ln}x\right)$$

F.O.I.L. (HAHA!!)

$$\Delta_{Bm} = \frac{1}{EI_B}\int_0^L \frac{F_H H^2}{n^2} - \frac{2F_H H^2}{Ln^2}x - \frac{2F_H H^2}{Ln^2}x + \frac{4F_H H^2}{L^2 n^2}x^2$$

$$= \frac{1}{EI_B}\int_0^L \frac{F_H H^2}{n^2} - \frac{4F_H H^2 x}{Ln^2} + \frac{4F_H H^2 x^2}{L^2 n^2}$$

$$= \frac{F_H H^2 x}{n^2} - \frac{4F_H H^2 x^2}{2L\cdot n^2} + \frac{4F_H H^2 x^3}{3L^2 n^2}\Bigg|_0^L$$

$$= \frac{F_H H^2 L}{n^2} - \frac{4F_H H^2 L^2}{2Ln^2} + \frac{4F_H H^2 L^3}{3L^2 n^2}$$

$$\Delta_{Bm} = 1 - \frac{4}{2} + 4/3 = \boxed{1/3 \times \frac{F_H H^2 L}{n^2 EI_B}} = \frac{1}{3}\times\frac{F_H H^2 L}{(2N_{col}-2)^2 EI_B}$$

Single Beam

So total Deflection is

$$\Delta_{Tot} = N_{\cancel{Ext col}\,\nearrow 2} \times \Delta_{colE} + N_{Int col}\times\Delta_{colI} + N_{BM}\times\Delta_{BM}$$